Phase locking of even and odd number of lasers on a ring geometry: effects of topological-charge

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Abstract: The effects of topological charge on phase locking an array of coupled lasers are presented. This is done with even and odd number of lasers arranged on a ring geometry. With an even number of lasers the topological-charge effect is negligible, whereas with an odd number of lasers the topological-charge effect is clearly detected. Experimental and calculated results show how the topological charge effects degrade the quality of the phase locking, and how they can be removed. Our results shed further light on the frustration and also the quality of phase locking of coupled laser arrays.

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References and links
1. Introduction

Phase locking of laser arrays has attracted considerable interest because it can lead to high output powers while maintaining good output beam quality and stability needed for medical, communications and industrial applications [1], and can serve as a powerful tool to investigate the behavior of coupled nonlinear oscillators and complex network dynamics [2]. Investigations of linear one-dimensional and two-dimensional coupled laser arrays have shown that the connectivity and geometry play an important role in determining the collective properties [3–6]. Theoretical investigations on the spatiotemporal dynamics and linear stability of coupled lasers on a ring geometry noted the effects of periodic boundary conditions, taking into account either even or odd number of lasers [7–9]. It was also shown that coupled lasers arranged in a Kagome geometry can induce frustration and thus no single minimal loss solution, which results in the poor phase locking quality [5]. In related investigations, it was found that waveguide arrays behave as photonic topological insulators in certain geometries [10]. Recently, a difference between the emission patterns from even and odd number of coupled vertical cavity surface emitting lasers (VCSELs) in a circular geometry was noted in a scenario in which the emission from each VCSEL is a single optical vortex with a unit topological charge [11].

In general, natural questions arise: are there any topological effects in the laser arrays? Can they influence phase locking of laser arrays? To respond to these questions, we investigated phase locking of an even number as well as odd number of negatively coupled lasers arranged on a ring geometry. Specifically, we exploited a degenerate cavity [12] to form a ring array of coupled lasers in which each laser emits a fundamental Gaussian mode. For the even number of negatively coupled lasers [5], with periodic boundary conditions, the only most probable and stable (minimal loss) solution is that in which the lasers satisfy the out-of-phase condition of $\pi$ phase shift between the adjacent lasers, i.e., $0, \pi, 0, \pi, 0, \ldots$. For the odd number, the lasers cannot satisfy this phase condition due to frustration by the periodic boundary conditions [13], but must still have nearly $\sim \pi$ phase difference between adjacent lasers in order to minimize...
the losses. In this case, phases of the lasers must circulate (or grow) from one laser’s site to the next in either a clockwise or anticlockwise direction. Namely, vortex and anti-vortex solutions, which are characterized by a non-zero phase circulation over a closed ring array that must be a multiple of $2\pi$. These are also referred to as topological charge solutions, where the topological charge of the vortex can be defined as \[14\]

$$TC = \frac{1}{2\pi} \sum_{i=1}^{N} \arg(E_i^* E_{i+1}),$$

with the argument taking the values $\arg(\Theta) \in [-\pi, \pi]$, and $E_i$ denotes the complex field of site $i$ on a ring array.

In this paper, we describe our investigations to determine the effects of topological-charge on phase locking of even and odd number of negatively coupled lasers arranged on a ring geometry. This was done by detecting and analyzing their near-field and far-field intensity distributions of first and second harmonics. The radial profiles of the far-field intensity distribution of second harmonic were then fitted with least squares method to extract the percentage of topological charges. To provide additional insight, we performed interference measurement, and also demonstrated how to remove the topological charge effects. Note that unlike negative coupling, positive coupling between the lasers can generate an in-phase stable solution $0, 0, 0, 0...$ that is not frustrated by the periodic boundary conditions of a closed ring, and hence is not inherently sensitive to topological charge effect.

2. Experimental arrangement

The experimental arrangement that was used in our experiments is schematically presented in Fig. 1. It includes of two main parts: (1) degenerate cavity laser for forming individual lasers on a ring geometry [12, 15], and (2) detection of the near-field and far-field intensity distributions of the output.

![Fig. 1. Experimental arrangement for forming even and odd number of lasers on a ring geometry, and detecting the near- and far-field intensity distributions. M1: rear mirror; d: quarter Talbot distance; L1 and L2: lenses; M2: front mirror; L3: imaging lens.](image)

The degenerate cavity part consisted of a Nd-YAG crystal gain rod of 1 cm diameter so it can support many laser channels, a high reflectivity rear mirror, a 95% reflectivity front mirror, and a mask of an array of apertures arranged on a ring, placed adjacent to the rear mirror. The lasing wavelength was 1064 nm. The gain medium was pumped by 100 $\mu$s pulsed xenon flash lamp operating at a repetition rate of 1 Hz. Two lenses $L_1$ and $L_2$ in a 4f configuration were placed inside the cavity so that any transverse electric field distribution at the mask plane is imaged onto the front mirror plane.

Accordingly, an array of individual lasers on a ring geometry is formed. The adjacent lasers in the array are coupled by diffraction, which is introduced by gradually shifting the rear mirror away from the mask so light diffracted from each laser is now coupled into its neighboring lasers. The strength and sign of coupling is determined by the distance between the rear mirror
and the mask. For a distance of $\frac{1}{4}$ of the Talbot length [16,17], the lasers are coupled negatively, and locked with a constant phase difference of $\pi$ radian. We also added a telescope and a KTP crystal to obtain the second harmonic of the output [18].

The detection part included lenses and a CCD camera for detecting the near-field and far-field intensity distributions of both the first and second harmonics. These were then analyzed to determine the effects of topological charge. To focus on the topological charge effects, the lasers were operated in the steady state regime (where dynamical effects [19–21] no longer exist) and the strength of coupling between the lasers was set well above the critical value [5, 22], so that all the lasers were locked in a stable (minimal loss) solutions.

3. Results and discussion

We experimentally detected and calculated the near-field and far-field intensity distributions of the first and second harmonics outputs when phase locking an array of even number (10) and odd number (11) of lasers arranged on a ring geometry. For the calculations, we assumed that the laser output intensities are equal, have Gaussian profiles, and out-of-phase or in-phase distributions when the lasers are phase locked. After arranging the even or odd number of Gaussian fields on ring geometry and performing a Fourier transform, we obtained the calculated far-field intensity distributions. The phases of the Gaussian fields were chosen in the out-of-phase configuration ($0, \pi, 0, \pi, ...$) for the even number of lasers, and in either the vortex or anti-vortex phase configuration ($+i \frac{5}{11} 2\pi$ or $-i \frac{5}{11} 2\pi$, where $i$ denotes the index of Gaussian field on the ring array) for the odd number of lasers. We verified numerically that these phase distributions are indeed stable stationary solutions of the Kuramoto model with nearest neighbor instantaneous coupling and parameters suitable for our experimental system [5].

The results are presented in Figs. 2 and 3. In our experiments the distance between the rear mirror and the mask plane was $\frac{1}{4}$ of the Talbot, so the lasers were negatively coupled. Specifically, adjacent lasers, shown in the near field distributions of Figs. 2(a) and 3(a), were out-of-phase or nearly out-of-phase. Thus, a dark spot is observed in the center of the far-field intensity distributions of the first harmonic, shown in Figs. 2(b) and 2(e) and Figs. 3(b) and 3(e), which indicates the out-of-phase locking. Since an array of 10 lasers has mirror symmetry, the first annular ring in the far-field intensity distribution of the first harmonic has ten spots. However, for an array of 11 lasers there is no mirror symmetry, so there are twenty-two spots in the first annular ring of the far-field intensity distribution of the first harmonic. We also measured the radial profiles of these intensity distributions of 10 and 11 lasers, and found that they are very similar.

A more significant difference was detected by resorting to second harmonic generation (SHG) which resulted in phase doubling [23], and recently exploited for converting out-of-phase coupled lasers to in-phase coupled lasers [18]. This is shown in the far-field intensity distributions of second harmonics for 10 and 11 lasers in Figs. 2(c) and 2(f) and Figs. 3(c) and 3(f), where the bright spots in the center denote in-phase locking. Here, there are very clear bright and dark annular rings in the distribution for 10 lasers, but not in that for 11 lasers. This obviously indicates that there is a significant difference in their phase locking solutions, that we then quantified by resorting to their radial intensity profiles, and extracted the topological charge effects on their phase locking.

As mentioned earlier, for an even number of negatively coupled lasers (e.g. 10), the most probable and stable (minimum loss) solution is when the lasers are out-of-phase, i.e., $0, \pi, 0, \pi, ...$. Other type of solutions, such as vortex and anti-vortex (topological charge) solutions, are very unlikely. However, for an odd number of lasers (e.g. 11), with periodic boundary conditions, an exact out-of-phase ($0, \pi, 0, \pi, ...$) solution is not possible. In this case, lasers can have two types of topological charge degenerate stable solutions: either $+i \frac{5}{11} 2\pi$ or $-i \frac{5}{11} 2\pi$. 
Fig. 2. Experimental and calculated intensity distributions when phase locking a ring array of 10 negatively coupled lasers. (a) experimental near-field; (b) experimental far-field first harmonic; (c) experimental far-field second harmonic; (d) calculated near-field; (e) calculated far-field first harmonic; (f) calculated far-field second harmonic.

where \( i \) denotes the index of laser on the ring array. Specifically, vortex and anti-vortex solutions, whereby the phase grows either in the clockwise direction or in the anticlockwise direction [24].

Figure 4 shows the radial profiles of the experimentally measured and calculated far-field intensity distributions of second harmonics for the 10 and 11 lasers given in Figs. 2(c) and 2(f) and Figs. 3(c) and 3(f). To determine the topological charge effects, we fitted the experimental radial intensity profiles to the calculated radial intensity profiles using least-squares method. The calculated radial intensity profiles contain a mixture of in-phase and topological charge solutions, and the percentage of these solutions was varied to achieve a good fitting. For the 10 lasers, the least-squares fitting indicated that the percentage of in-phase is very close to 100% and that of topological charge is very close to 0%. This confirms that the most stable solution for negatively coupled 10 lasers is \( 0, \pi, 0, \pi, \ldots \). For the 11 lasers, the least-squares fitting indicated that the in-phase solution is \( \sim 50\% \) and the topological charge solution is \( \sim 50\% \) (25% vortex and 25% anti-vortex). In both cases (even and odd lasers) the first three lobes fit nicely, but not the fourth lobe (see also Figs. 2 and 3). This can be attributed to the fact that in the experiments the shapes of the laser outputs were not perfectly circular as assumed for the calculations.

Since 11 lasers cannot satisfy an exact out-of-phase \( (0, \pi, 0, \pi, \ldots) \) solution, there should not be a bright spot in the center of the far-field intensity distribution when doubling the lasers phases by SHG. But yet, we experimentally detected a bright spot due to \( \sim 50\% \) in-phase intensity distribution. This is due to the fact that each laser has many longitudinal modes [5], each of which randomly contributes a vortex or anti-vortex phase configuration. Consequently, as a result of sum frequency generation in the SHG, the summation of two longitudinal modes with vortex and with anti-vortex phase configurations (50% of the sums) produces an in-phase
Fig. 3. Experimental and calculated intensity distributions when phase locking a ring array of 11 negatively coupled lasers. (a) experimental near-field; (b) experimental far-field first harmonic; (c) experimental far-field second harmonic; (d) calculated near-field; (e) calculated far-field first harmonic; (f) calculated far-field second harmonic.

Fig. 4. The radial profiles of the far-field intensity distributions of the second harmonics. (a) 10 lasers; (b) 11 lasers. The solid red curves denote the experimentally measured radial profiles from Figs. 2(c) and 3(c). The solid blue curves denote the calculated radial profiles from Figs. 2(f) and 3(f) for 0% and 50% topological charge solutions, respectively, that best fit to the experimental data.
intensity distribution. The other 50% are sums of two vortex or two anti-vortex phase configurations that produce the topological charge solution, in excellent agreement with our experimental results [18].

The results above indicate that with 10 lasers only one stable solution is obtained, whereas with 11 lasers two topological charge solutions, namely, vortex and anti-vortex are obtained. A simultaneous vortex and anti-vortex solution leads to frustration in phase locking of 11 lasers.

We also performed experiments to determine the effects of array size and boundary conditions. The experiments were performed with 10 and 11 lasers in closed boundary, 30 and 31 lasers in closed boundary, and 9 and 10 lasers in open boundary. The results of their far-field radial intensity profiles are presented in Fig. 5. Figure 5(a) shows that the 10 and 11 lasers have different radial intensity profiles, which indicates different phase locked solutions, as explained earlier. For the large arrays of 30 and 31 lasers, the radial intensity profiles are essentially identical, as shown in Fig. 5(b). As the interaction between the lasers in these arrays is short ranged, so there is no coherence between very distant lasers, in accordance to the Mermin-Wagner theorem [25]. Thus the restricting periodic boundary conditions on the phase configurations for the even and odd number of lasers no longer apply in these larger arrays, so the same radial profiles are obtained. Figure 5(c) shows the case when one laser is blocked in the arrays of 10 and 11 lasers. This turns the closed ring arrays into open chains (open boundary condition), so the lasers have the out-of-phase configuration. Accordingly, blocking of one laser in the 10 lasers array has no effect on phase locking, but blocking of one laser in the 11 lasers array removes the topological charge effects, so the two arrays produce almost identical radial intensity profiles. The results of Fig. 5 verify that topological charge effects are significant only when restricting periodic boundary conditions exist.

In order to provide further insight about the effect of topological charge on phase locking, we performed another experiment where we detected and analyzed the interference between the lasers. Specifically, the role of topological charge on phase locking was determined by measuring the visibility of the fringes in the interference pattern and also the relative phases between the lasers. Figure 6 shows schematically the arrangement for analyzing the phase locking between the lasers. It includes a Mach–Zehnder interferometer, where at the first beam splitter the laser beam splits into two parts that propagate along two different channels of the interferometer. In one channel the output from the lasers is imaged onto the CCD camera. In the other channel, a beam from a single laser is selected, using a pinhole of size 50 µm, and then expanded in order to overlap with all lasers at CCD camera. Thus, a single laser interferes with itself and with all the other lasers. The interference pattern between the lasers is recorded by CCD camera. We also calculated the visibility as a function of the distance between a reference single laser and all other lasers. The calculations were based on the interference between two Gaussian fields of equal amplitudes and the known phase difference between them, as described.
earlier. For the odd number of lasers, the two interference patterns corresponding to vortex and anti-vortex phase configurations were added to calculate the average visibility.

Figure 7 shows the interference results for ring arrays of negatively coupled 10 and 11 lasers. For the 10 lasers, the fringe visibility is almost uniform, as shown in Figs. 7(a) and 7(b). However, for the 11 lasers the fringe visibility drops significantly on either side of reference laser, and reaches a minimum at the farthest distance from it, as shown in Figs. 7(d) and 7(e). This is more clearly evident in the experimentally analyzed and calculated fringe visibility, shown in Figs. 7(f) and 7(g). For the 10 lasers, there is only a single (out-of-phase) stable solution, as is also evident in the phase analysis Fig. 7(b), so the fringe visibility is essentially constant. For the 11 lasers, there are two (vortex and anti-vortex) degenerate stable solutions. These two solutions produce two different interference patterns having the same radial intensity profiles, but shifted in phase relative to each other. The shifting in their phases is maximum at the farthest distance from the reference laser. Thus, averaging their interference patterns (over the many longitudinal modes) yields the lowest fringe visibility at the farthest distance from the reference laser, in excellent agreement with our experimental results.

We repeated this analysis when one of the lasers in the arrays of 10 and 11 lasers is blocked. The results are presented in Fig. 8(a) and 8(d). As noted earlier, the blocking of one laser changes the closed ring array into an open chain array of 9 and 10 lasers, respectively. Removing the periodic boundary restrictions enables a single out-of-phase stable solution in both cases, as clearly evident in the analyzed phase patterns shown in Figs. 8(b) and 8(e). The interference patterns for 9 and 10 lasers, and their fringe visibilities, compared in Fig. 8(f), are similar with uniform visibility, indicating that there are no topological charge effects for the open chain array.

4. Concluding remarks

To conclude, we have shown the effects of topological charge on phase locking of negatively coupled lasers arranged on a close ring geometry. In particular, we found that for an even number of lasers, there is negligible topological charge effect, and a single stable solution. However, for an odd number of lasers there are topological charge effects, and two (vortex and anti-vortex) stable solutions. Simultaneous occurrence of these two solutions leads to frustration and poor
Fig. 7. Interference results for a closed ring of 10 and 11 lasers. (a) and (b) experimental interference pattern and corresponding analyzed phase pattern for 10 lasers; (d) and (e) experimental interference pattern and corresponding analyzed phase pattern for 11 lasers; (c) color bar for the phase pattern; (f) and (g) experimental and calculated fringe visibility between all lasers and the reference laser (denoted by 0). The calculation assumes perfect coherence between the lasers. This explains the small difference from the experimental visibility but confirms a dramatic difference between even and odd number of lasers on a closed ring.
Fig. 8. Interference results for open ring of 9 and 10 lasers. (a) and (b) experimental interference pattern and corresponding analyzed phase pattern for 9 lasers; (d) and (e) experimental interference pattern and corresponding analyzed phase pattern for 10 lasers; (c) color bar for the phase patterns; (f) experimental fringe visibility between all lasers and the reference laser (denoted by 0) for the 9 and 10 lasers, indicating that without restricting boundary conditions (open chain) they are similar.

Phase locking. Increasing the number of lasers on the ring beyond the phase locking range (∼ 20 lasers in our system) or cutting the closed ring by blocking one of the lasers removes the restricting periodic boundary conditions, and thus removes the effects of topological charge even for an odd number of lasers. Our results shed further light on the frustration and also the quality of phase locking of coupled laser arrays. Furthermore, our investigations could be related to synchronization and cluster formation in ring laser networks [26–28], where the effects of even and odd number of lasers were noted [12].

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